Affleck-Dine leptogenesis via multiscalar evolution in a supersymmetric seesaw model

Masato Senami[1] and **Tsutomu Takayama**[2]

ICRR, University of Tokyo, Kashiwa 277-8582, Japan

Abstract

A leptogenesis scenario in a supersymmetric standard model extended with introducing right-handed neutrinos is reconsidered. Lepton asymmetry is produced in the condensate of a right-handed sneutrino via the Affleck-Dine mechanism. The LH_u direction develops large value due to a negative effective mass induced by the right-handed sneutrino condensate through the Yukawa coupling of the right-handed neutrino, even if the minimum during the inflation is fixed at the origin. The lepton asymmetry is nonperturbatively transfered to the LH_u direction by this Yukawa coupling.

[1]senami@icrr.u-tokyo.ac.jp. Now Department of Micro Engineering, Kyoto University.
[2]tstkym@icrr.u-tokyo.ac.jp

1 Introduction

The origin of the baryon asymmetry of the universe is one of unsolved problems. The existence of the baryon asymmetry is confirmed in several ways. Among them, the observation of the cosmic microwave background constrains the amount of the baryon asymmetry with considerable accuracy. It is given in terms of the baryon-to-entropy ratio as [1]

$$\frac{n_B}{s} = (8.7 \pm 0.3) \times 10^{-11} \tag{1}$$

where s is the entropy density of the universe.

The origin of neutrino masses is another problem of the standard model (SM). By observation of neutrino oscillations [2, 3], it is confirmed that at least two flavors of neutrinos have non-zero masses. On the other hand, the sum of masses of three flavors of neutrinos is constrained as $\sum m_\nu < 2\,\mathrm{eV}$ by cosmological observations [1, 4]. Hence, the SM should be extended to explain nonzero neutrino masses and the smallness of them. Introducing heavy right-handed Majorana neutrinos provides a good explanation for the problem of neutrino masses via the seesaw mechanism [5].

Heavy right-handed Majorana neutrinos also provide an attractive solution of the origin of the baryon asymmetry by leptogenesis [6]. In the leptogenesis scenario, lepton asymmetry is generated at first, and then the sphaleron process partially transfers it into the baryon asymmetry. In the thermal leptogenesis scenario, the lepton asymmetry is generated via the out-of-equilibrium decay of heavy right-handed Majorana neutrinos produced in the primordial thermal bath [6, 7]. Therefore, this scenario requires the reheating temperature T_R after the inflation to be higher than the mass of the lightest right-handed neutrino. However, $T_R \lesssim 10^8\,\mathrm{GeV}$ is required to avoid the overproduction of the gravitino in supersymmetric (SUSY) theories [8]. The production of the sufficient lepton asymmetry is very difficult in SUSY models due to this constraint [7].

Hence, several alternative leptogenesis scenarios have been considered in SUSY theories. The Affleck-Dine mechanism [9, 10, 11], which produces asymmetry between particle and antiparticle in the condensate of a scalar field, is one of interesting scenarios, since many species of scalar particles exist in SUSY models. In the Affleck-Dine leptogenesis along the LH_u flat direction [12], the source of lepton number violation originates in the operator of the neutrino mass. However, in the SUSY seesaw model, if the mass of a right-handed neutrino is smaller than the Hubble parameter at the end of the inflation H_{inf}, the LH_u direction is not flat due to the F-term potential

from the Yukawa coupling of the right-handed neutrino[1]. The right-handed neutrino mass should be less than about $10^{13}\,\mathrm{GeV}$ for $m_\nu \sim 1\,\mathrm{eV}$ if the Yukawa coupling is less than the unity. For the chaotic inflation, which predicts $H_{\rm inf} \sim 10^{13}\,\mathrm{GeV}$, it is severe that the right-handed neutrino masses dominate over $H_{\rm inf}$.

In addition to right-handed neutrinos, there are right-handed sneutrinos $\tilde{N}$ in the SUSY seesaw model. The potential for $\tilde{N}$ is flat except for SUSY mass terms, if R-parity is conserved. Therefore, quantum fluctuation of a right-handed sneutrino can be large at the end of the inflation [12], as long as the right-handed neutrino mass is smaller than $H_{\rm inf}$. Hence, the right-handed sneutrino can have the large number density at the end of the inflation. Thus, if CP-violation in the $\tilde{N}$ decay into leptons and anti-leptons is large enough, sufficient lepton asymmetry can be produced.

Later, this scenario was drastically changed after SUSY breaking in the early universe was reported [14]. Due to the negative Hubble mass square of a right-handed sneutrino, minima of the potential of the right-handed sneutrino is largely deviated from the origin during the inflation. Therefore, the right-handed sneutrino has large value. After the inflation, this scalar field condensate begins coherent oscillations. If there exists CP-violation in the potential, e.g. B-term of $\tilde{N}$, this condensate acquires the particle number asymmetry for the right-handed sneutrino via the Affleck-Dine mechanism. In Ref. [15], one scenario along with multidimensional Affleck-Dine mechanism [16] is reported. The asymmetry of $\tilde{N}$ is nonperturbatively transferred to the LH_u direction, if the LH_u direction is approximately flat, that is, both $\tilde{N}$ and LH_u directions have large value during the inflation. In this scenario, the lepton asymmetry can be generated without CP-violation in the right-handed sneutrino decay.

On the other hand, in Ref. [17], Allahverdi and Drees reported that the particle number asymmetry of $\tilde{N}$ can be transfered by perturbative decay of $\tilde{N}$ even if the LH_u direction is not approximately flat or has a positive Hubble mass square, i.e., without the evolution of the scalar field along the LH_u direction. Because of the Majorana nature of the right-handed neutrino, right-handed sneutrinos decay with generating ± 1 lepton number in the same decay rate if SUSY is conserved. These two decay rates are deviated from each other because of SUSY breaking by thermal effects in the early universe. Therefore, non-zero lepton number can be generated in

[1] If a neutrino Yukawa coupling is very tiny, e.g. in the Dirac neutrino model, the LH_u direction is approximately flat and can have large value during inflation. The Affleck-Dine leptogenesis in the Dirac neutrino model can explain the baryon asymmetry without the lepton number violation except for the sphaleron process [13].

compensation for some parameter tuning.

However, we found in this work that the evolution of the LH_u direction is induced by a negative effective mass given by the right-handed sneutrino condensate, even if the LH_u direction is not approximately flat or has a positive Hubble mass square. Therefore, the evolution of the LH_u direction cannot be neglected in broad parameter region. Thus, the evolution of the scalar fields and the lepton asymmetry is complicated, like the scenario in Ref. [15]. In this paper, we reconsider the scenario discussed in Ref. [17], including the evolution of the LH_u direction. We will see that the lepton asymmetry in the right-handed sneutrino condensate can be nonperturbatively transfered to the LH_u direction condensate via the interaction between these scalar fields. We also summarize the condition that the evolution of the LH_u direction is relevant.

The rest of this paper is organized as follows. In the next section, we summarize the set-up of this scenario. In Section 3, we discuss the evolution of scalar fields. In Section 4, we consider the evolution of the lepton asymmetry. The resultant baryon asymmetry is discussed in Section 5. We also comment on the difference between this scenario and the previous one. Finally, we summarize the result in Section 6.

2 Model

We consider the SUSY seesaw model. The superpotential is given by

$$W = W_{\rm MSSM} + y_\nu N L H_u + \frac{M_N}{2} N N + \frac{\lambda}{4 M_{\rm Pl}} N N N N, \tag{2}$$

where $W_{\rm MSSM}$ is the superpotential of the Minimal SUSY SM (MSSM), y_ν is the coupling between right-handed neutrinos and left-handed leptons, and M_N is the mass matrix of right-handed neutrinos. We included non-renormalizable superpotential of N with coupling constant given by λ, which is important for successful baryogenesis in this scenario. For simplicity, only one flavor of right-handed sneutrino is considered. This can be naturally realized by assuming that three neutrino masses satisfies the condition, $M_N = M_{N_1} < H_{\rm inf} < M_{N_2} < M_{N_3}$. Hence, we consider only one flavor of left-handed leptons coupling to the right-handed sneutrino. We choose y_ν and M_N to be real and positive by redefining superfields. We assign lepton number -1 to $\tilde{N}$ although right-handed neutrinos cannot carry $U(1)$ charge because of their Majorana nature. The lepton number of $\tilde{N}$ is violated by B-terms and non-renormalizable terms, but this violation is suppressed by

SUSY breaking effects or cut-off scale $M_{\rm Pl}$. Therefore, we can assign the lepton number to $\tilde{N}$ in this work.

A light neutrino mass is given by the seesaw mechanism as,

$$m_\nu = \frac{y_\nu^2 v^2}{M_N} \sim 10^{-2}\,{\rm eV}\left(\frac{y_\nu}{10^{-2}}\right)^2\left(\frac{10^{11}{\rm GeV}}{M_N}\right). \tag{3}$$

Here $v \sim \mathcal{O}(100)\,{\rm GeV}$ is the vacuum expectation value (vev) of H_u.

We consider the evolution of the right-handed sneutrino $\tilde{N}$ and the LH_u direction parametrized by a complex scalar field ϕ, namely,

$$\tilde{L} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi\\0\end{pmatrix}, H_u = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\\phi\end{pmatrix}. \tag{4}$$

The LH_u direction is D-flat, but not F-flat due to the Yukawa coupling of the neutrinos. Nevertheless, the evolution of this direction gives an important effect on the leptogenesis. Hence, in this work we take this direction into consideration.

Including Hubble-induced SUSY-breaking effects and thermal-corrections, the scalar potential is given by

$$\begin{aligned}
V(\phi,\tilde{N}) \quad = \quad & \frac{y_\nu^2}{4}|\phi|^4 + M_N^2|\tilde{N}|^2 + y_\nu^2|\phi|^2|\tilde{N}|^2 + \frac{\lambda^2}{M_{\rm Pl}^2}|\tilde{N}|^6 \\
& + \left[\left(\frac{y_\nu}{2}M_N\phi^2\tilde{N}^* + \frac{y_\nu\lambda}{2M_{\rm Pl}}\phi^2\tilde{N}^{*3} + \frac{\lambda M_N}{M_{\rm Pl}}\tilde{N}\tilde{N}^{*3}\right) + h.c.\right] \\
& + c_\phi H^2|\phi|^2 - c_N H^2|\tilde{N}|^2 \\
& + \left[\left(\frac{bH}{2}M_N\tilde{N}^2 + \frac{a_y y_\nu}{2}H\phi^2\tilde{N} + \frac{a_\lambda\lambda}{4M_{\rm Pl}}H\tilde{N}^4\right) + h.c.\right] \\
& + V_{\rm th}(\phi),
\end{aligned} \tag{5}$$

where H is the Hubble parameter and we ignored low-energy SUSY-breaking terms, which are not relevant to this leptogenesis. The first and second lines are the F-term potential from the superpotential (2), while the third and fourth lines are Hubble-induced SUSY breaking terms. Here we assumed that the Hubble-induced mass of ϕ is positive in order to set it at the origin during the inflation even for small y_ν, while that of $\tilde{N}$ is negative in order to give large value. Therefore, real parameters $c_\phi \sim 1$ and $c_N \sim 1$ are both assumed to be positive. Complex parameters a_y, a_λ and b determine magnitude of the Hubble-induced A-terms and B-term, respectively. Absolute

values of these constants are typically $\mathcal{O}(1)$ during the inflation. However, since these terms are generally indued by coupling with the inflaton, they oscillate rapidly by the oscillation of the inflaton after the inflation. Thus, effective values of these terms vanish if they are averaged over a time scale much longer than the period of the oscillation of the inflaton. Therefore, we simply assume $a_y = a_\lambda = b = 0$ after the inflation. The fifth line is thermal-mass corrections [18]. These are given by

$$V_{\rm th}(\phi) \equiv \sum_{f_k|\phi|<T} c_k f_k^2 T^2 |\phi|^2. \tag{6}$$

Here, f_k denotes coupling constants of left-handed leptons or up-type Higgs, and c_k are determined by degrees of freedom of these particles. The temperature of the thermal plasma T before the reheating ends is estimated as

$$T \sim \left(g_*^{-\frac{1}{2}} H T_R^2 M_{\rm Pl} \right)^{\frac{1}{4}}, \tag{7}$$

where $g_* \simeq 200$ is the effective total degree of freedom of the thermal bath and $M_{\rm Pl} \simeq 2.4 \times 10^{18}\,\mathrm{GeV}$ is the reduced Planck mass. There are also thermal-log corrections [19], but they do not have dominant effects in this leptogenesis.

During the inflation, the potential of $\tilde{N}$ has minima at $|\tilde{N}| \simeq \sqrt{H M_{\rm Pl}/\lambda}$. However, starting from these minima, $\tilde{N}$ is eventually trapped at minima of F-term potential, $|\tilde{N}| \simeq \sqrt{M_N M_{\rm Pl}/\lambda}$. In order to avoid this disastrous consequence, we assume that $\tilde{N}$ is in a multiplet of the $SO(10)$ Grand Unified Theory (GUT). When $|\tilde{N}| > M_{\rm GUT}$, the steep D-term potential of the GUT gauge group appears. Therefore, the initial value of $\tilde{N}$ is given by $|\tilde{N}_{\rm ini}| = M_{\rm GUT} = 10^{16}\,\mathrm{GeV}$. In addition, it is required that the local maximum of the F-term potential of the radial direction of $\tilde{N}$, which is given by $|\tilde{N}| \simeq \sqrt{M_N M_{\rm Pl}/(3\lambda)}$, is placed at $|\tilde{N}| > M_{\rm GUT}$. This requirement can be rewritten by

$$M_N/\lambda > 1.2 \times 10^{14}\,\mathrm{GeV}. \tag{8}$$

Provided with these conditions, $\tilde{N}$ rolls down towards the origin and oscillates around there after $H < M_N$.

3 Evolution of scalar fields

The evolution equations of the scalar fields are given by

$$\ddot{\tilde{N}} + (3H + \Gamma_N)\dot{\tilde{N}} + \frac{\partial V}{\partial \tilde{N}^*} = 0, \tag{9}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0, \tag{10}$$

where $\Gamma_N = y_\nu^2 M_N/(4\pi)$ is the decay width of $\tilde{N}$. The initial condition of $\tilde{N}$ is $|\tilde{N}_{\rm ini}| = M_{\rm GUT}$. On the other hand, the initial phase of $\tilde{N}$ is dependent on whether or not the Hubble parameter $H_{\rm inf}$ is much larger than the effective mass of the phase direction during inflation. The potential is dominated by A or B-term under the condition (8). If the effective mass of the phase direction is much smaller than $H_{\rm inf}$, $\theta_{\tilde{N}}$ is randomly displaced from minima of the potential. Otherwise, $\theta_{\tilde{N}}$ may be fixed at one of minima of the potential. In both cases, the successful baryogenesis can be realized as we will see later. In the former case, $H_{\rm inf}$ is constrained to be $H_{\rm inf} < 3 \times 10^{12}$GeV for small enough baryonic isocurvature perturbation as discussed in Section 5.2. In the latter case, large M_N or λ is required to derive the large effective mass of the phase direction. Meanwhile, ϕ has positive Hubble-induced mass and large effective mass $y_\nu M_{\rm GUT}$ via the neutrino Yukawa coupling. Therefore, ϕ is trapped at the origin $\phi = 0$ and only has quantum fluctuation suppressed by its large effective mass. Although the value $\langle\phi\rangle$ averaged over the universe is expected to vanish, a finite value $\langle\phi^2\rangle$ remains.

3.1 Before destabilization of ϕ

For $H > M_N$, $\tilde{N}$ remains as $|\tilde{N}| = |\tilde{N}_{\rm ini}|$. When H becomes $H < M_N$, the mass term dominates the evolution of $\tilde{N}$ and $\tilde{N}$ begins oscillation around the origin. The amplitude of the oscillation decreases with

$$|\tilde{N}| \sim M_{\rm GUT}\frac{H}{M_N}. \tag{11}$$

The cross term in the F-term potential ($y_\nu M_N \phi^2 \tilde{N}^* + h.c.$) gives negative contribution to the effective mass of ϕ. Therefore, when H decreases to H_1, ϕ becomes tachyonic and acquires large amplitude because the mass squares from $H^2|\phi|^2$, $y_\nu^2|\phi|^2|\tilde{N}|^2$ and the cross term decreases as H^2, H^2 and H, respectively. The negative mass dominates over the Hubble induced mass when $H < y_\nu M_{\rm GUT}$, while it dominates over the effective mass from

the quartic coupling when $H < M_N^2/(y_\nu M_{\rm GUT})$. The LH_u direction ϕ becomes tachyonic when both conditions are satisfied. Therefore, we define the Hubble parameter at the destabilization H_1 as

$$H_1 = \begin{cases} y_\nu M_{\rm GUT} & \left(m_\nu < 10^{-8}\,{\rm eV}\left(\frac{M_N}{10^{11}\,{\rm GeV}}\right)\right) \\ \dfrac{M_N^2}{y_\nu M_{\rm GUT}} & \left(m_\nu > 10^{-8}\,{\rm eV}\left(\frac{M_N}{10^{11}\,{\rm GeV}}\right)\right) \end{cases}. \tag{12}$$

Note that always $H_1 < M_N$. The negative contribution from $\phi^2 \tilde{N}^{*3}$ term cannot be dominant under the condition (8).

Thermal-mass terms give positive contributions to the effective mass of ϕ, while thermal-log corrections are ineffective because ϕ does not have large value. The destabilization of ϕ is prevented if thermal-mass corrections dominate over the negative mass contribution. Otherwise, the same scenario as discussed in Ref. [17] is realized. This condition is given by

$$H_1 > H_1' \simeq 10^7\,{\rm GeV}\left(\frac{g_*}{200}\right)^{-\frac{1}{2}}\left(\frac{T_R}{10^9{\rm GeV}}\right)^2\left(\frac{y_\nu}{10^{-2}}\right)^{-2}\left(\frac{M_{\rm GUT}}{10^{16}{\rm GeV}}\right)^{-2}, \tag{13}$$

where we consider the thermal-mass correction from top (s)quarks in the thermal bath.

In addition, the right-handed sneutrino must not decay before the destabilization. This constraint is given as $\Gamma_N < H_1$. If $H_1 = y_\nu M_{\rm GUT}$, this is rewritten by $M_N < 10^{16}\,{\rm GeV}(m_\nu/10^{-2}\,{\rm eV})^{-1/3}$. On the other hand, for $H_1 = M_N^2/(y_\nu M_{\rm GUT})$, this constraint is given as $M_N < 10^{16}\,{\rm GeV}(m_\nu/10^{-2}\,{\rm eV})^{-3}$, Thus, this constraint is ignorable in both cases.

In Ref. [17], it is also discussed that the parametric resonance [20, 21] may cause fast decay of $|\tilde{N}|$. Since $\tilde{N}$ oscillates with large amplitude, it gives large oscillating effective mass of ϕ, unless $\tilde{N}$ has a circle-like trajectory in complex plain due to large angular momentum. This may result in exponential amplification of fluctuation of ϕ and rapid decrease of the amplitude of the $\tilde{N}$ oscillation. If $\tilde{N}$ decays via the parametric resonance before the destabilization of ϕ, the destabilization cannot take place. However, this is not serious for $m_\nu > 10^{-8}\,{\rm eV}(M_N/10^{11}\,{\rm GeV})$, because ϕ can decay into other particles. The typical amplification rate under broad parametric resonance is estimated to be proportional to $\exp(0.175 M_N t)$ [21]. On the other hand, the decay rate Γ_ϕ has the smallest value just before the destabilization of ϕ, $\Gamma_\phi \sim \sum g_i^2 M_N/(8\pi)$, where g_i are Yukawa and gauge couplings, and we sum over all final states. Hence, the parametric resonance is safely negligible

since $\Gamma_\phi > 0.175 M_N$ in the MSSM. On the other hand, parametric resonance may take place for $m_\nu < 10^{-8}\,\mathrm{eV}(M_N/10^{11}\,\mathrm{GeV})$, because $\Gamma_\phi > 0.175 M_N$ is not guaranteed. Since the detail of the parametric resonance is involved, we do not discuss whether the parametric resonance actually gives fast decay of $|\tilde{N}|$. For simplicity, we neglect the case $m_\nu < 10^{-8}\,\mathrm{eV}(M_N/10^{11}\,\mathrm{GeV})$.

3.2 After destabilization of ϕ

After $H = H_1$, the potential of ϕ has two distinct minima in opposite phase directions,

$$|\phi| \sim \sqrt{\frac{2M_N|\tilde{N}(H_1)|}{y_\nu}} \sim \sqrt{\frac{2M_{\mathrm{GUT}}H_1}{y_\nu}}. \tag{14}$$

The initial condition at the destabilization is determined by quantum fluctuation during the inflation and the subsequent evolution. After the destabilization of ϕ, long wavelength modes of fluctuation of ϕ get large value via tachyonic instability [22, 23]. Once the value of ϕ begins to track the minimum of the potential, it can be interpreted as a classical field in a local patch of the universe. Hereafter we assume that the universe consists of patches in which condensate of ϕ has various initial values. Hence, the resultant lepton asymmetry is estimated by averaging over results from various values of initial quantum fluctuation. Since a typical comoving wavelength of this quantum fluctuation is negligibly small compared with the present horizon scale, this fluctuation has no cosmologically observable consequence. In other words, the initial fluctuation of ϕ averaged over cosmological scale is suppressed to be negligibly small, because such scale is far larger than the horizon scale at the epoch $H = H_1$. Indeed, the comoving length k_1^{-1} of the latter scale is estimated to be

$$k_1^{-1} \sim \mathcal{O}(10)\,\mathrm{km} \times \left(\frac{H_1}{10^8\mathrm{GeV}}\right)^{-\frac{1}{3}} \left(\frac{T_R}{2\times 10^6\mathrm{GeV}}\right)^{-\frac{1}{3}} \left(\frac{g_*}{100}\right)^{-\frac{1}{12}}. \tag{15}$$

Note that the problem of domain walls separating two minima is not serious since they disappear when ϕ evaporates.

With the minimization of the cross term, the dominant contributions in this epoch can be rewritten by

$$V_F(\phi, \tilde{N}) = \left(\frac{y_\nu}{2}|\phi|^2 - M_N|\tilde{N}|\right)^2 + y_\nu^2|\phi|^2|\tilde{N}|^2. \tag{16}$$

Though this potential has the global minimum at $|\phi| = |\tilde{N}| = 0$, there is a valley on the trajectory $y_\nu|\phi|^2 = 2M_N|\tilde{N}|$, lifted by $y_\nu^2|\phi|^2|\tilde{N}|^2$. Therefore,

ϕ and $\tilde{N}$ oscillate around the origin approximately satisfying the relation, $y_\nu|\phi|^2 = 2M_N|\tilde{N}|$. The amplitudes of $|\phi|$ and $|\tilde{N}|$ decrease proportionally to $H^{1/2}$ and H, respectively.

The condensate of $\tilde{N}$ decays by the neutrino Yukawa coupling at $H \sim \Gamma_N$. Since the decay rate acts as a friction term in the evolution equation, $\tilde{N}$ is strongly fixed at the minimum of the potential soon after $H = \Gamma_N$. Hence, the evolution of the scalar fields is determined by that of ϕ. After $|\tilde{N}|$ is integrated out by $|\tilde{N}| = y_\nu|\phi|^2/(2M_N)$, the effective potential of ϕ is given by

$$V_{\rm eff}(\phi) \simeq \frac{y_\nu^4}{4}\frac{|\phi|^6}{M_N^2} + V_{\rm th}(\phi), \tag{17}$$

where we included thermal potentials. The LH_u direction does not decay before $\tilde{N}$ does, since all decay modes with coupling constants larger than y_ν are kinematically forbidden as long as $H > \Gamma_N$. Since the first term decreases as H^6, thermal-corrections usually dominate the evolution of ϕ, soon after the $\tilde{N}$ decay starts. Afterward, ϕ oscillates around the origin and eventually evaporates.

In Fig. 1, we show an example of the evolution of the scalar fields. The black (grey) line show the evolution of $|\phi|(|\tilde{N}|)$. Here, parameters are taken as $M_N = 10^{11}$ GeV, $y_\nu = 10^{-2}$, $T_R = 2 \times 10^6$ GeV, $c_\phi = c_N = 1$ and $\lambda = 10^{-4}$. We assumed $a_y = a_\lambda = b = 0$ as mentioned above. As the initial condition, we assumed $|\tilde{N}_{\rm ini}| = M_{\rm GUT}$ ($\arg \tilde{N}_{\rm ini} = \pi/4$) and $|\phi_{\rm ini}| = 10^{11}$ GeV. Here we evaluated typical value of quantum fluctuation by averaging over the horizon scale after the inflation with $H_{\rm inf} = 10^{13}$ GeV, because the destabilization of ϕ is not instantaneous. In order to take the randomness of the initial condition into account, we iterated the same calculation for various initial amplitudes and phases of ϕ. This figure is the result for $\arg(\phi_{\rm ini}) = 0$. It can be seen that $|\tilde{N}|$ is fixed at $|\tilde{N}| = M_{\rm GUT}$ until $H \simeq M_N/3$ and begins oscillation at $H \simeq M_N/3$ from this figure. We can also confirm that the destabilization of ϕ completes at $H \sim H_1 = 10^8$ GeV. Afterward, $|\phi|$ decreases proportionally to $H^{1/2}$, while $|\tilde{N}|$ does proportionally to H. Finally, $\tilde{N}$ decays at $H \sim \Gamma_N$.

4 Evolution of lepton asymmetry

The evolution of the lepton asymmetry is determined by the evolution of the scalar fields. We define lepton asymmetries in the condensate of the

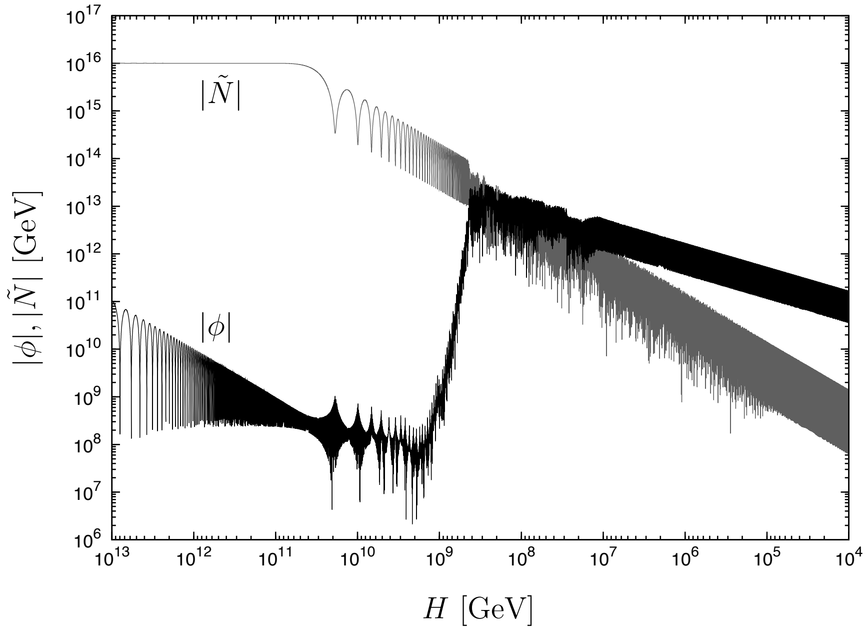

Figure 1: The evolution of the values of $|\phi|$ (black) and $|\tilde{N}|$ (grey) as a function of H. We take $M_N = 10^{11}$ GeV, $y_\nu = 10^{-2}$, $T_R = 2 \times 10^6$ GeV and $\lambda = 10^{-4}$.

right-handed sneutrino $L_{\tilde{N}}$ and in that of the LH_u direction L_ϕ as follows,

$$L_{\tilde{N}} \equiv -i(\dot{\tilde{N}}^* \tilde{N} - \dot{\tilde{N}} \tilde{N}^*), \tag{18}$$

$$L_\phi \equiv \frac{i}{2}(\dot{\phi}^* \phi - \dot{\phi} \phi^*). \tag{19}$$

Note that we assigned lepton number -1 to $\tilde{N}$. Evolution equations of these lepton asymmetries are given from the evolution equations of these scalar fields, Eqs. (9) and (10),

$$\begin{aligned} \frac{d}{dt}\left(\frac{L_{\tilde{N}}}{H^2}\right) + \Gamma_N \frac{L_{\tilde{N}}}{H^2} & \\ = -\frac{1}{H^2}\Bigg[y_\nu M_N \mathrm{Im}(\phi^{*2}\tilde{N}) &+ \frac{4\lambda M_N}{M_{\rm Pl}} \mathrm{Im}\left(\tilde{N}^3 \tilde{N}^*\right) \\ &+ \frac{3 y_\nu \lambda}{M_{\rm Pl}} \mathrm{Im}\left(\phi^{*2}\tilde{N}^3\right)\Bigg], \end{aligned} \tag{20}$$

$$\frac{d}{dt}\left(\frac{L_\phi}{H^2}\right) = -\frac{1}{H^2}\left[y_\nu M_N \mathrm{Im}(\phi^{*2}\tilde{N}) + \frac{y_\nu \lambda}{M_{\rm Pl}} \mathrm{Im}\left(\phi^{*2}\tilde{N}^3\right)\right], \tag{21}$$

where Hubble-induced phase-dependent terms are dropped as mentioned above. The right-hand side of these equations are source terms of the lepton asymmetry. If a dominant source term is simply scaling with $t^\gamma \propto H^{-\gamma}$ on an average, where the index γ is a constant, the evolution of the lepton asymmetry is simple: if $\gamma > -3$, the lepton asymmetry increases proportionally to $t^{\gamma+3}$, while if $\gamma < -3$, the lepton asymmetry is fixed. In the case of $\gamma = -3$, the lepton asymmetry increases proportionally to $\log t$. Thus, the lepton asymmetry is also considered to be almost fixed in this case.

The evolution of these lepton asymmetries are very complicated. However, the evolution of the left-right asymmetry $L_\phi - L_{\tilde{N}}$ is simpler. The evolution of $L_\phi - L_{\tilde{N}}$ is given by the equation,

$$\frac{d}{dt}\left(\frac{L_\phi - L_{\tilde{N}}}{H^2}\right) = \frac{1}{H^2}\left[\frac{4\lambda M_N}{M_{\rm Pl}}{\rm Im}\left(\tilde{N}^3\tilde{N}^*\right) + \frac{2y_\nu\lambda}{M_{\rm Pl}}{\rm Im}\left(\phi^{*2}\tilde{N}^3\right)\right] + \Gamma_N\frac{L_{\tilde{N}}}{H^2}. \quad (22)$$

Thus, the dominant contribution from $\phi^{*2}\tilde{N}$ term to the left-right asymmetry cancels. Since the other terms attenuates sufficiently fast, the evolution of the left-right asymmetry is fixed after $\tilde{N}$ begins oscillation.

4.1 Before destabilization of ϕ

In this era, the decay width is negligible because $H \gg \Gamma_N$. The second and third terms in the right-hand side of Eq. (22) are also negligible, since the fluctuation of ϕ is suppressed by its large effective mass. Thus, the first term is dominant because the phase direction of $\tilde{N}$ is generally displaced from minima determined by $\tilde{N}^3\tilde{N}^*$ term. Therefore, $L_\phi - L_{\tilde{N}}$ is determined only by the evolution of $\tilde{N}$. The magnitude of left-right asymmetry $L_\phi - L_{\tilde{N}}$ becomes fixed when $\tilde{N}$ begins oscillation, because the amplitude of the source term scales with H^4. The amount of the left-right asymmetry at $H = M_N/3$ is estimated by

$$\frac{|L_\phi - L_{\tilde{N}}|}{s'} \simeq \frac{|L_{\tilde{N}}|}{s'} \sim \frac{6\lambda M_{\rm GUT}^4 T_R}{M_N^2 M_{\rm Pl}^3}\delta_{\rm eff}, \quad (23)$$

where the entropy parameter s' is defined as $s' \equiv 4M_{\rm Pl}^2H^2/T_R$, and $\delta_{\rm eff} \lesssim 1$ is the phase factor. Note that s' is normalized by the entropy after the reheating completes.

4.2 After destabilization of ϕ

After ϕ acquires large value, it takes part in the evolution. In Eqs. (20) and (21), the first term of the right-hand side becomes dominant. Since this term induces rapid exchange between $L_{\tilde{N}}$ and L_ϕ, these asymmetries oscillate rapidly.

However, $L_\phi - L_{\tilde{N}}$ is fixed because all the source terms for left-right asymmetry attenuate with H^4 after ϕ gets large value. At the epoch ϕ is rolling down to the displaced minimum, the contribution to the $L_\phi - L_{\tilde{N}}$ from the second term of Eq. (22) is difficult to estimate because of the fast and non-linear evolution. Furthermore, this contribution depends on the initial phase of random fluctuation of ϕ. However, we confirmed that this contribution is subdominant if the contribution from the first term is sufficiently large, since the contribution from the second term almost vanishes on an average over cosmological scale, which contains large number of patches of horizon scale at the epoch $H = H_1$. In other words, the initial fluctuation of ϕ averaged over cosmogical scale is suppressed to be negligibly small, because such scale is far larger than the horizon scale at the epoch $H = H_1$. After $H \sim \Gamma_N$, $\tilde{N}$ decays rapidly by the neutrino Yukawa coupling. The leading decay channels of $\tilde{N}$ are $\tilde{N} \to H_u \tilde{L}$ and $\tilde{N} \to \tilde{H}_u \bar{L}$, creating lepton numbers $+1$ and -1, respectively. Therefore, the lepton asymmetry stored in the condensate of $\tilde{N}$ is not transfered to the SM sector if CP-violation in the decay is small. On the other hand, L_ϕ is fixed because the source term decreases rapidly after $H \sim \Gamma_N$. The amount of the fixed asymmetry of L_ϕ is the same order of $L_\phi - L_{\tilde{N}}$. The precise amount is dependent on the phase of the oscillation of L_ϕ and difficult to estimate analytically. Therefore, We define ϵ as the fraction of $L_\phi - L_{\tilde{N}}$ inherited to L_ϕ. The fixed asymmetry of L_ϕ also depends on the initial value of ϕ. Since the value of $L_\phi - L_{\tilde{N}}$ is determined only by the evolution of $\tilde{N}$, non-vanishing asymmetry is inherited in L_ϕ after averaging results for various initial values of ϕ. Eventually, L_ϕ is released to the SM sector as the condensate ϕ evaporates.

The fact that the final lepton asymmetry have non-vanishing value can also be understood qualitatively by the following discussion. The potential of ϕ has two distinct minima after destabilization due to coupling with $\tilde{N}$. The crucial observation is that the direction of the rotational evolution of $\tilde{N}$ is determined only by initial evolution of $\tilde{N}$, therefore this direction is the same over the cosmological scale. Since $\tilde{N}$ rotates in a definite direction, these two minima also rotate in one direction. Because the rotational motion of ϕ is induced by this rotation of two minima, ϕ rotates in one direction whichever is the minimum that ϕ is trapped. After $\tilde{N}$ decays, the direction

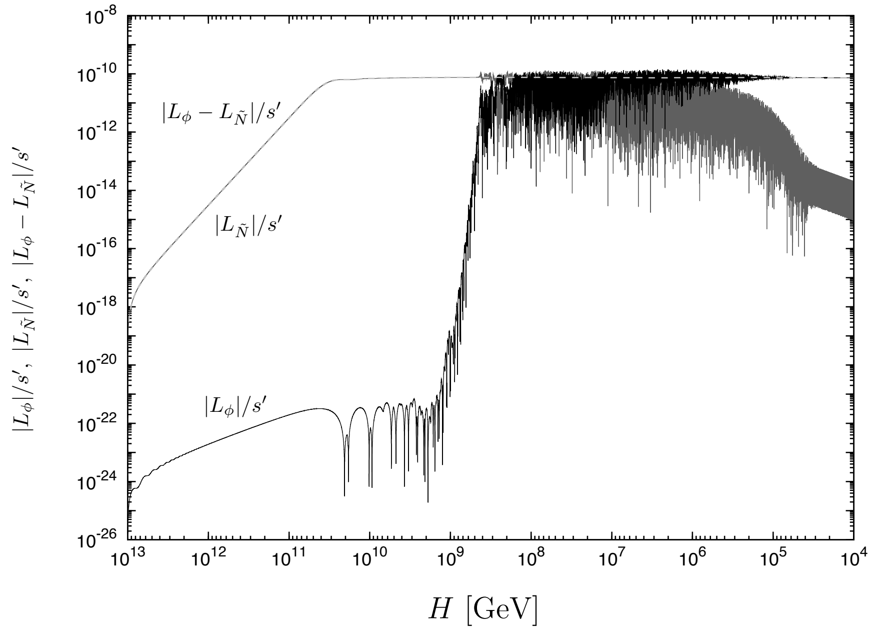

Figure 2: The evolution of lepton asymmetries $|L_\phi|/s'$ (solid black), $|L_{\tilde{N}}|/s'$ (solid grey) and the left-right asymmetry $|L_\phi - L_{\tilde{N}}|/s'$ (dashed grey). Since these asymmetries oscillate, we show magnitudes of them. The parameters are the same as for Fig. 1.

of the rotation of ϕ does not change because of approximate conservation of the angular momentum. Thus, the final direction of the rotation of ϕ, which is equivalent to the sign of the final lepton asymmetry L_ϕ, is the same all over the universe.

We show the evolution of asymmetries in Fig. 2 for the same parameter set as we chose for Fig. 1. The solid black line, the solid grey line, and the dashed grey line indicate evolutions of $|L_\phi|/s'$, $|L_{\tilde{N}}|/s'$ and $|L_\phi - L_{\tilde{N}}|/s'$, respectively. It can be seen that for $H > M_N$, $|L_{\tilde{N}}|$ increases, while $|L_\phi|$ is negligible. At $H \sim H_1$, ϕ receives a fraction of the asymmetry of $\tilde{N}$. We can also confirm from this figure that for $H < \Gamma_N \simeq 10^6\,\mathrm{GeV}$, $L_{\tilde{N}}$ decreases exponentially and L_ϕ is fixed. For the parameter used in our calculation, the estimation using Eq. (23) gives $|L_\phi|/s = 8.3 \times 10^{-11}\delta_{\mathrm{eff}}\epsilon$. We iterated the same calculation for 25 various initial phases, and the result is $|L_\phi|/s = 5.2 \times 10^{-11}$ on an average. This implies $\delta_{\mathrm{eff}}\epsilon \sim 0.6$ in this case. We also confirmed that other choices of initial amplitude $|\phi_{\mathrm{ini}}|$ do not change

the result.

5 Resultant baryon asymmetry

5.1 Estimation of baryon asymmetry

The lepton asymmetry released into the SM sector is transfered to the baryon asymmetry through the sphaleron process by the ratio $n_B = (8/23)n_L$ [24], where n_B is the resultant baryon asymmetry in the thermal bath and n_L is the lepton asymmetry produced by this scenario. Hence, the baryon asymmetry is estimated as

$$\frac{n_B}{s'} \sim \frac{48}{23}\frac{\lambda M_{\rm GUT}^4 T_R}{M_N^2 M_{\rm Pl}^3}\delta_{\rm eff}\epsilon. \tag{24}$$

In Fig. 3, we show the parameter region in which the right amount of baryon asymmetry is produced and some constraints on the M_N-T_R plane. Solid lines with dotted parts show region in which resultant baryon asymmetry (24) is $n_B/s = 8.7 \times 10^{-11}$ for various λ. We assumed that $\delta_{\rm eff}\epsilon = 1$. The baryon asymmetry is simply proportional to this factor. Dotted parts of these lines are excluded because of the constraint (8). Dashed lines indicate the constraint that thermal-corrections must not dominate over the negative mass contribution of ϕ for several neutrino masses (see Eqs. (12) and (13)). Note that neutrino masses are determined by Eq. (3). We can see that this constraint is not stringent. Above these lines, the scenario discussed in Ref. [17] can be realized. In addition, if initial $\theta_{\tilde{N}}$ is not fixed at any minima determined by Hubble-induced A- or B-term, $M_N < H_{\rm inf} < 3 \times 10^{12}$GeV is required in order that baryonic isocurvature perturbation should be sufficiently small (see Section 5.2). This result indicates that successful baryogenesis via this scenario favors larger M_N and higher T_R for large λ and smaller M_N and lower T_R for small λ.

As we can see in Eq. (24), the resultant baryon asymmetry is proportional to $|\tilde{N}_{\rm ini}|^4 = M_{\rm GUT}^4$. If $\tilde{N}$ is the multiplet of the subgroup of SO(10) GUT, e.g. SU(2)$_R$, $|\tilde{N}_{\rm ini}| = M_{SU(2)_R} < M_{\rm GUT}$ should be used. Unless λ is extremely small, it is difficult to explain the origin of baryon asymmetry by this scenario for $|\tilde{N}_{\rm ini}| = M_{SU(2)_R} < M_{\rm GUT}$.

Finally, we summarize the difference between our scenario and that in Ref. [17]. The latter scenario may be realized above dashed lines in Fig. 3. Our scenario can explain the baryon asymmetry for lower T_R than that in Ref. [17]. On the other hand, the scenario in Ref. [17] has an advantage that the sufficient baryon asymmetry can be generated if $|\tilde{N}_{\rm ini}| = M_{SU(2)_R} <$

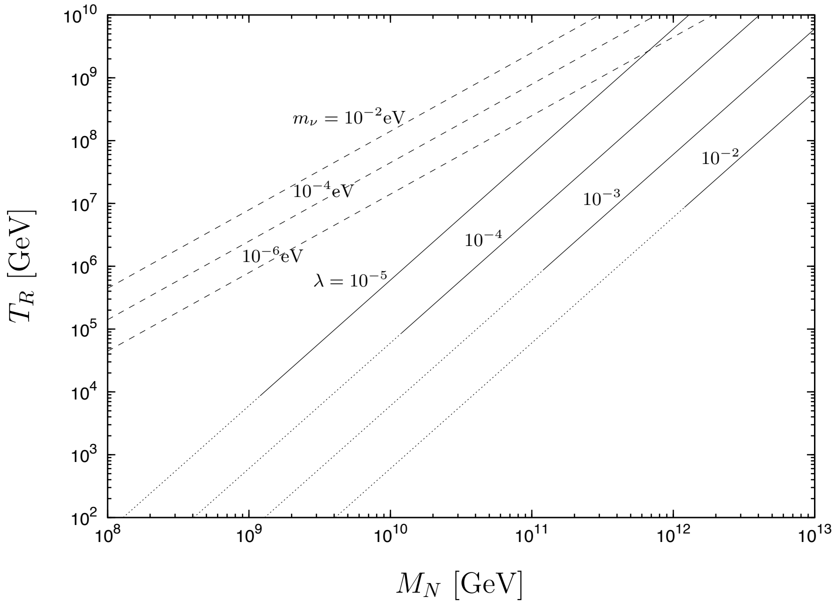

Figure 3: Solid lines show parameter region in which the rusultant baryon asymmetry is $n_B/s = 8.7 \times 10^{-11}$ for various values of λ. We assumed that $\delta_{\rm eff}\epsilon = 1$. Dotted parts of these lines are excluded by the condition $M_N/\lambda > 1.2 \times 10^{14}$ GeV. Dashed lines show the constraint $H > H_1'$ for several m_ν, which is not stringent. Above these lines, the scenario discussed in Ref. [17] can be realized.

$M_{\rm GUT}$. Another advantage of their scenario is that non-renormalizable terms in the superpotential is not required. On the other hand, the most important advantage of our scenario is that the successful baryogenesis can be realized without fine-tuning. The scenario in Ref. [17] requires some tuning between the decay width of Γ_N and the magnitude of B-terms. However, any parameter tuning is not required in our scenario.

5.2 Baryonic isocurvature perturbation

In the case that the potential of the phase direction $\theta_{\tilde{N}}$ of $\tilde{N}$ is sufficiently flat during the inflation, $\theta_{\tilde{N}}$ is randomly displaced from minima of the potential. Then, $\theta_{\tilde{N}}$ has isocurvature perturbation,

$$\delta\theta_{\tilde{N}} = \frac{H_{\rm inf}}{\sqrt{2k^3}|\tilde{N}|}. \tag{25}$$

for Fourier mode k. Since $L_{\tilde{N}}$ is produced via the displacement of $\theta_{\tilde{N}}$ from minima determined by $\tilde{N}^3\tilde{N}^*$ term, this isocurvature perturbation results in the isocurvature perturbation of $L_{\tilde{N}}$, which is finally transfered to the isocurvature perturbation of baryon asymmetry. Since $n_B \propto \delta_{\rm eff}$ and $\delta_{\rm eff}$ can be estimated by $\delta_{\rm eff} \sim \sin(2\delta\theta_{\tilde{N}})$, the amplitude of the baryonic isocurvature perturbation can be estimated to be

$$\frac{\delta n_B}{n_B} \sim 2\delta\theta_{\tilde{N}}. \tag{26}$$

According to the constraint on the baryonic isocurvature perturbation in terms of the ratio between the power spectrum of matter isocurvature perturbation and that of curvature perturbation [25],

$$B_a \equiv \sqrt{\frac{\mathcal{P}_\mathcal{S}}{\mathcal{P}_\mathcal{R}}} = \sqrt{\frac{1}{2.4\times 10^{-9}}}\sqrt{\frac{k^3}{2\pi^2}\frac{\Omega_b^2}{\Omega_m^2}\left\langle\frac{\delta n_B^2}{n_B^2}\right\rangle} < 0.31, \tag{27}$$

the Hubble paraneter during the inflation is constrained to be $H_{\rm inf} < 3\times 10^{12}$ GeV. Note that this constraint can be avoided if initial $\theta_{\tilde{N}}$ is fixed at one of minima determined by Hubble-induced A- or B-term. This can be realized if the value of b or a_λ during the inflation satisfies the following condition, $bM_N \sim H_{\rm inf}$ or $\lambda a_\lambda > (H_{\rm inf}/10^{14}\,{\rm GeV})$.

6 Summary

We reconsidered the leptogenesis scenario in the SUSY seesaw model with including the evolution of the LH_u direction. We found that the LH_u direction acquires large value due to a negative effective mass induced by the right-handed sneutrino condensate through the Yukawa coupling, even if the minimum of ϕ is fixed at the origin during the inflation, unless the reheating temperature T_R is sufficiently high. The lepton asymmetry is first produced in the condensate of $\tilde{N}$ via the Affleck-Dine mechanism, then transfered nonperturbatively to the condensate of ϕ by the Yukawa coupling. This lepton asymmetry is released into the baryon asymmetry in the SM sector by the sphaleron process. In this scenario, the appropriate amount of the baryon asymmetry can be generated for low T_R avoiding the gravitino problem without parameter tuning.

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